the distributions of velocity and pressure are seen to approach those for the case of a sphere in an ideal fluid. But as $b/a \to 0$, i.e. as the drop is flattened out by the aerodynamic forces in a direction perpendicular to that of the undisturbed flow, the velocity and pressure are seen to deviate considerably from those of a sphere. For example, a typical empirical value[5] of the eccentricity parameter, corresponding to the condition of maximum droplet diameter, is $b/a \simeq 0.35$.

The general observation can be made that drop flattening has the dual effect of reducing the magnitude of the flow velocity over much of the windward surface while at the same time increasing the fluid velocity in the region of the equator. It is felt that the significant increase in shearing velocity near the equator is an important contributing factor in the high-speed aerodynamic breakup of liquid droplets. In other words, it is apparent that, by virtue of the droplet deformation or flattening, a high shearing velocity is generated in the vicinity of the equator, which hastens the disintegration process by enhancing the boundary-layer stripping of liquid away from the droplet equatorial surface. Also, the reduction of flow velocity over a large portion of the windward surface has the effect of producing an extensive quasi-stagnation region, as shown in Fig. 2, and the extent of this region is seen to depend on the magnitude of b/a. Hence, the pressure coefficient over much of the surface is positive ($C_p \geq 0$), the manifestation of which is a higher drag force than otherwise would exist if the droplet shape remained spherical. Since Taylor's model does not account for either of these important effects, it seems reasonable to conclude that it will give conservative results for liquid droplet acceleration and disintegration rate in a high-speed gas stream.

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Nonequilibrium in an Arc Constrictor

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SINCE the first construction of a cascade arc in which an electrical discharge is confined by the cooled wall of a tube, numerous theories have been developed to predict the associated flow condition. Despite inherent differences, the theories have all had in common the assumption of local equilibrium. In recent years, however, evidence has appeared which suggests that this assumption is inappropriate in regions adjacent to the confining wall.[1,2] The objective of this Note is to obtain a first estimate of the nature and the extent of nonequilibrium in a typical arc. This is done by delineating and uncoupling the effects that influence the equilibrium state of an arc plasma, assigning an appropriate time scale to each effect, and comparing relative magnitudes for typical conditions.

Possible nonequilibrium effects of special interest in an arc include ionization nonequilibrium, nonequipartition of translational energy between electrons and heavy particles, and the existence of an electron velocity distribution function which is non-Maxwellian. At the same time that certain processes act to promote these nonequilibrium effects, competing processes act to restore the corresponding equilibrium condition. For example, the external electric field, thermal conduction, ambipolar diffusion, and inelastic recombination collisions all promote nonequipartition of translational energy. In contrast, electron-heavy particle elastic collisions provide the dominant mechanism for maintaining equipartition. Similarly, the external field and inelastic collisions act to promote a non-Maxwellian electron velocity distribution, while self-collisions in the electron subgas act to restore the equilibrium distribution. Finally, ambipolar diffusion acts to promote ionization nonequilibrium at the same time that ionization or recombination processes act to maintain the corresponding equilibrium condition. In the core of the arc, where diffusion depletes the local electron concentration, the ionization rate must be sufficiently large to maintain an equilibrium electron number density; in the peripheral arc region, diffusion increases the local electron concentration and a sufficient recombination rate is needed to maintain equilibrium.

In this paper the aforementioned equilibrium departure and restoration mechanisms are uncoupled, and each is characterized by an appropriate time scale. Time-scale calculations are then performed using temperature and species concentration profiles obtained from an equilibrium theory for the asymptotic region of a 1 cm diam, atmospheric argon arc.[3] The calculations are performed for arc currents of 35 and 600 amps and provide an approximate estimate of the extent of nonequilibrium in the arc. In addition, a simple treatment of two competing mechanisms is used to illustrate the possible existence of nonequipartition in arcs for a range of operating conditions.

The form of several of the time scales of interest may be obtained from the use of phenomenological arguments in connection with the general relaxation equation

$$\partial Q/\partial t = (1/\tau)(Q^* - Q) \qquad (1)$$

where Q may be either a species temperature or concentration and Q^* is generally the corresponding equilibrium value. The so-called equipartition time τ_{eq} is defined as the time required to reach a 67% reduction in some initial electron-heavy particle temperature difference due to elastic electron-atom and electron-ion collisions. To determine this time, the general expression for the random species energy exchange rate in a nonequipartition plasma[4] is used in connection with appropriate gas-kinetic cross sections for electron-ion[5] and electron-atom[6] collisions. Similarly, a characteristic ohmic heating time τ_{oh} may be defined as the time required for the temperature of the electron subgas to be elevated by a factor of e through application of the external field. The electrical conductivity required for the calculations was evaluated using the mixture rule suggested by Kruger and Viegas.[7] The characteristic reaction time τ_r is defined as the time required to achieve a factor of e change in the electron concentration through either collisional ionization or recombination. Since the time-scale calculations are performed using equilibrium temperature and concentration profiles,

Received May 21, 1970. This work was done at NASA Ames Research Center. The work of F. P. Incropera was supported by a summer ASEE-NASA Research Fellowship. This support is gratefully acknowledged.

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the recombination and ionization times are identical. The form of the reaction time is determined using the two-step ionization model with appropriate cross-section data.[4,8]

The electron self-collision time $\tau_{c,ee}$ is a measure of the effect of coulombic electron self-collisions in promoting a Maxwellian velocity distribution for that species; it is calculated using the expression developed by Spitzer.[9] The characteristic flow time is the ambipolar diffusion time, and it is calculated in two ways. To determine a diffusion time τ_{dif}, which is a measure of the effect of diffusion on nonequipartition, a radial ambipolar diffusion velocity V_{amb} is computed using the development suggested by Camac and Kemp,[10] and is divided into a characteristic length over which a 10% reduction in temperature occurs. A second diffusion time τ'_{dif}, which is more appropriate to chemical considerations, is suggested by the electron continuity equation

$$\tau'_{dif} = \left| \frac{rn_e}{(\partial/\partial r)(rn_eV_{amb})} \right| \quad (2)$$

where r is the radial coordinate and n_e is the electron concentration. Finally, although conduction is a significant nonequipartition mechanism, the energetics of the asymptotic arc dictate that it be of lesser importance than the ohmic heating effect.

The results of the aforementioned calculations are presented in Figs. 1 and 2 for 35 and 600 amp argon arcs. The dashed part of the curve for τ'_{dif} corresponds to the region where Eq. (2) is singular as a result of the net flux of electrons to a differential volume element changing sign. Using a double arrowed symbol ($\leftrightarrow$) here to separate characteristic times associated with the equilibrium restoration mechanism from its corresponding equilibrium departure mechanism(s), the comparisons relevant to the types of nonequilibrium mentioned previously are as follows: electron velocity distribution: $\tau_{c,ee} \leftrightarrow \tau_{oh}, \tau_r$, nonequipartition: $\tau_{eq} \leftrightarrow \tau_{oh}, \tau_r, \tau_{dif}$, and ionization nonequilibrium: $\tau_r \leftrightarrow \tau'_{dif}$.

From these epochal considerations, it is apparent that there exists a central core for which the assumption of thermal equilibrium is a valid approximation, but that nonequilibrium effects are significant in the arc periphery. The chemical

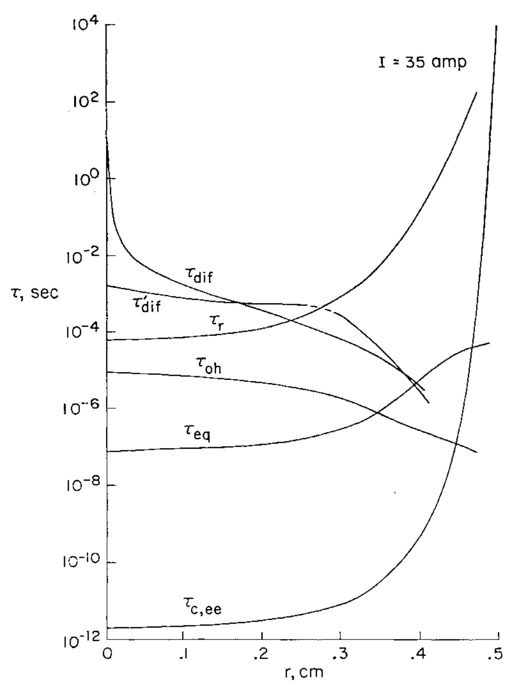

Fig. 1 Time scales for equilibrium departure and restoration mechanisms for the 35-amp argon arc as a function of arc radius.

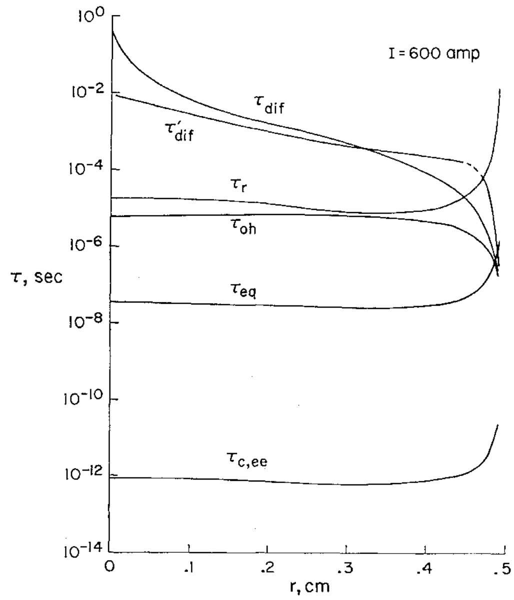

Fig. 2 Time scales for equilibrium departure and restoration mechanisms for the 600-amp argon arc as a function of arc radius.

nonequilibrium effect dominates, with the nonequipartition effect assuming a lesser role. Departure from a Maxwellian velocity distribution for the electron subgas will occur, if at all, only at the lower arc currents. Clearly, the equilibrium core contracts with decreasing arc current, thereby increasing the spatial extent of the nonequilibrium condition.

From Figs. 1 and 2 there appears to be a substantial portion of the arc (especially at low currents) in which nonequipartition might result primarily from an imbalance between the ohmic heating and the elastic energy-transfer mechanism for electrons. In steady state these energy transfer mechanisms can be related by Eq. (23) of Ref. 11. This equation is for a spatially uniform steady-state plasma in which the electrons and heavy species are assumed to have Maxwellian distributions at temperatures T_e and T, respectively. While not describing the spatial energetics of an arc exactly, this expression can be assumed to hold locally at any arc radius. This energy equation has been used, with the conductivity given by Eq. (16) of Ref. 11 and the cross sections from Ref. 6, to evaluate the local gas temperature T of an argon plasma as a function of the local electron temperature T_e, the local pressure, and the strength of the applied electric field. The species number densities were calculated from the Saha equation for argon with the assumption of ionization equilibrium at the electron temperature and Dalton's law of partial pressures for given values of the total pressure and the gas temperature. The results of this calculation are given in Fig. 3.

It is apparent from Fig. 3 that nonequipartition effects increase with the applied voltage for fixed gas temperature and pressure. As the pressure increases for a fixed voltage and gas temperature, the nonequipartition effects decrease reflecting the fact that the electrons have more frequent elastic energy exchange collisions with the heavy particles. As the electron temperature increases for a fixed voltage and gas pressure, nonequipartition effects diminish (to nearly zero for low voltages) and then increase again.

This minimum temperature difference results from the electron temperature dependence of the coulombic collision frequency between electrons and ions and is not affected significantly by the presence of multiple ionized species. It is expected that thermal conduction or nonelastic effects would be important at the higher electron temperatures of Fig. 3 and cause T_e and T to coalesce more than shown. Even though the energy equation used in the present analysis was incom-

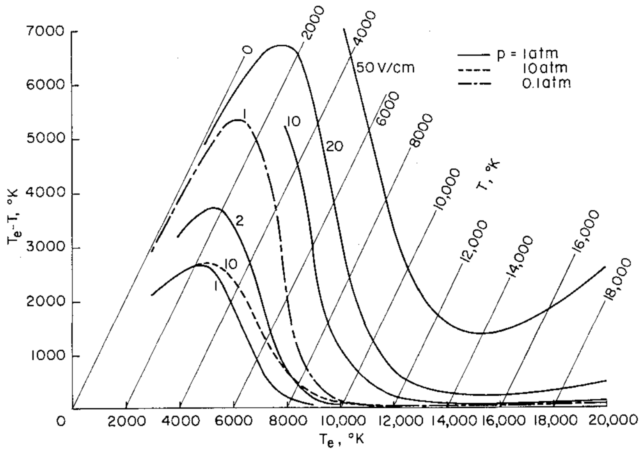

Fig. 3 Nonequipartition effects in an argon plasma illustrated by lines of constant electric field strength as a function of electron temperature T_e and gas temperature T for various pressures.

plete, the results suggest that for arc plasmas in which the dominant electron energy-transfer mechanisms are ohmic heating and elastic collisions, nonequipartition effects should be expected over a wide range of arc parameters.

In a recent study, Kruger,[12] using a two-temperature ambipolar diffusion model, analyzed the nonequilibrium behavior of a confined arc plasma at atmospheric pressure. The conclusions derived from the present study are in good qualitative agreement with the results of Kruger's calculations.

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